EIT forward computation based on element-free Galerkin method for hematoma detection

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Element-free Galerkin method (EFGM) is applied to the computation of EIT forward problem. It's hopeful to overcome the defect of finite element method (FEM) for modeling tiny volume objects, such as hematomas within the organ. In this paper, the basic principle and implementation of EFGM are studied with simulation tests and obtained results.

***Index Terms*—Element-free Galerkin method (EFGM), EIT, forward problem, hematoma detection**

I. Introduction

THE hematomas are very harmful to human body, and delayed diagnosis can cause tragic cases like disability and mortality [1]. The early detection of hematomas becomes necessary and inevitable. However, the hematomas in early stage always have tiny volumes and the distributions of the area are sometimes flat and narrow, which make it difficult to detect by existing conventional medical imaging system. With the advantages of non-invasive, sensitive and cheap, electrical impedance tomography (EIT) technology may be a good choice to detect the high resistivity of hematomas.

In this paper, element-free method (EFM) is applied to compute EIT forward problem [2]. EFM is also called meshless method. As a special case of weighted residual method, EFM only needs node information rather than element information. It's hopeful to overcome the defect of traditional finite element method (FEM) in modeling the hematomas with tiny volumes. Among many forms of EFM, element-free Galerkin method (EFGM) [3]-[4] is widely used. EFGM can reduce the error caused by the local approximation of field function in FEM. The simulation results show the good performance of EFGM.

II. Method

In EFGM, background mesh is necessary to approximate the realistic potential distribution by moving least square method (MLSM) [5].

MLSM is an interpolation polynomial, the field function is set as $u(x)$, and

$$u^h(x) = \sum_{j=1}^{m} p_j(x) a_j(x) \equiv p^{\mathrm{T}}(x) a(x) \tag{1}$$

where, $p_j(x)$ is the basis function related to spatial dimension and superposition times, $a_j(x)$ is the unknown coefficient matrix. Generally, the estimation points and the data points are not exactly the same, as shown in Fig.1, where $\hat{u}_i$ is the field function at the point x_i, and u_i is the computation result from equation (1).

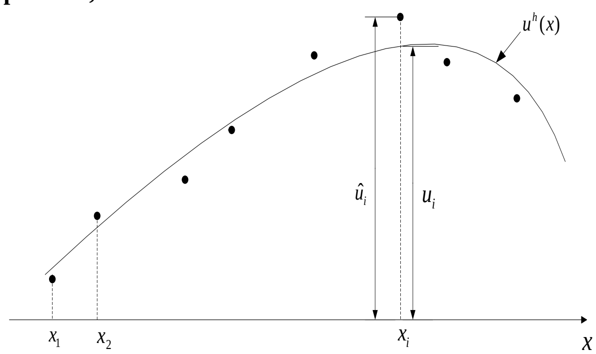

Fig. 1. The difference between $\hat{u}_i$ and u_i .

Put $a_j(x)$ into $u^h(x)$, and we can get

$$u^h(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_j(x) [A^{-1}(x) B(x)]_{ji} \hat{u}_i \tag{2}$$

where,

$$A(x) = \sum_{i=1}^{n} w_i(x) p(x_i) p^{\mathrm{T}}(x_i) \tag{3}$$

$$B(x) = [w_1(x) p(x_1), w_2(x) p(x_2), \cdots, w_n(x) p(x_n)] \tag{4}$$

$$w_i(x) \equiv w(x - x_i) \tag{5}$$

where, $w_i(x)$ is the weighted function, and the distance is the only variable in it. Restrict the basis nodes related to an interpolation node in a circle surrounding with the center of the interpolation node, which is called support domain.

In EFGM, the chosen weighted function needs to be non-zero in the influence domain of the node, but be zero out of the influence domain. This is called the local properly of the shape function. The continuity of the weighted function determines the continuity of the shape function, and determines the smoothness of the filed function [6]. The selection of different weighted functions will affect the accuracy of calculation and convergence of the workload. Some typical weighted functions are shown in Fig.2.

In addition, the spline weighted function is a continuous and differentiable function in the strict sense. It's necessary to select the reasonable parameters to guarantee the function approximation. For the sake of simplicity, we select the quartic spline with the advantages of smoothness, strict sense of the guide function with C^2 continuity, and simple implementation in sectional form.

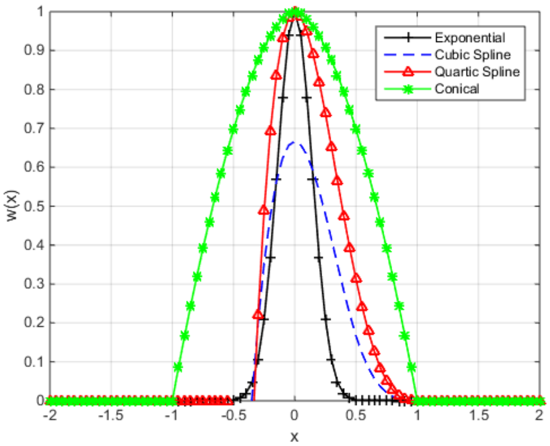

Fig. 2. Typical weighted functions.

The quartic spline function is shown as follows

$$w(s)=\begin{cases}1-6s^2+8s^3-3s^4, & s\le 1\\ 0, & s>1\end{cases} \tag{6}$$

where, $s=r_i/r_{mi}$, $r_i=\|x-x_i\|$, and r_{mi} is the radius of influence.

The integral equilibrium equation is formed by the Gauss integral equation for high algebraic accuracy.

In EFGM, the whole space is divided into a background grid which is independent with the nodes. The background is only for integral computation, and has nothing on the building of the approximate function. The Gauss integration nodes are created in each grid, and only the Gauss nodes in the calculation domain are needed to be considered.

III. Simulation and Results

The 2D normalized circle domain is used to simulate the internal hematomas in the organs. There are 16 electrodes at the edge of the circle. In Fig. 3, we suppose the normal tissue as blue dots and the hematoma organization as red asterisks, which represent an electrical impedance abnormal domain, such as lesion, at the lower right side of the circle.

Based on the forward computation results by EFGM, the EIT image is reconstructed by node back projection algorithm (NBPA) [7]. Fig. 4 illustrates that the position of the hematoma silhouette is at the lower right side of the whole circle domain obviously.

IV. Conclusion

In EFGM, element subdivision is not needed. For MLSM, the curve fitting is ever smoother, and the Gauss integral has higher precision. From the simulation result we can see that EFGM has the ability to compute EIT forward problem accurately, especially for tiny object, and the reconstruction result has the proper and accurate location.

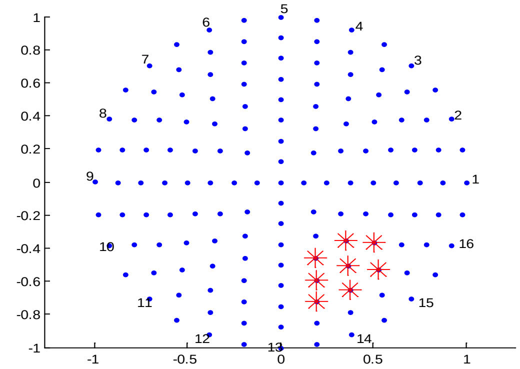

Fig. 3. Simulation circle domain for EFGM.

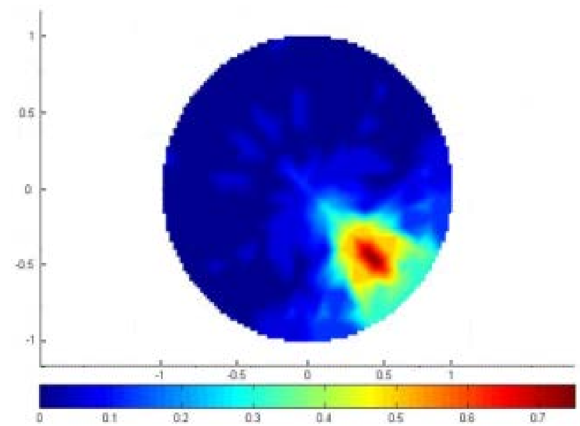

Fig. 4. EIT image reconstructed based on EFGM.

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